

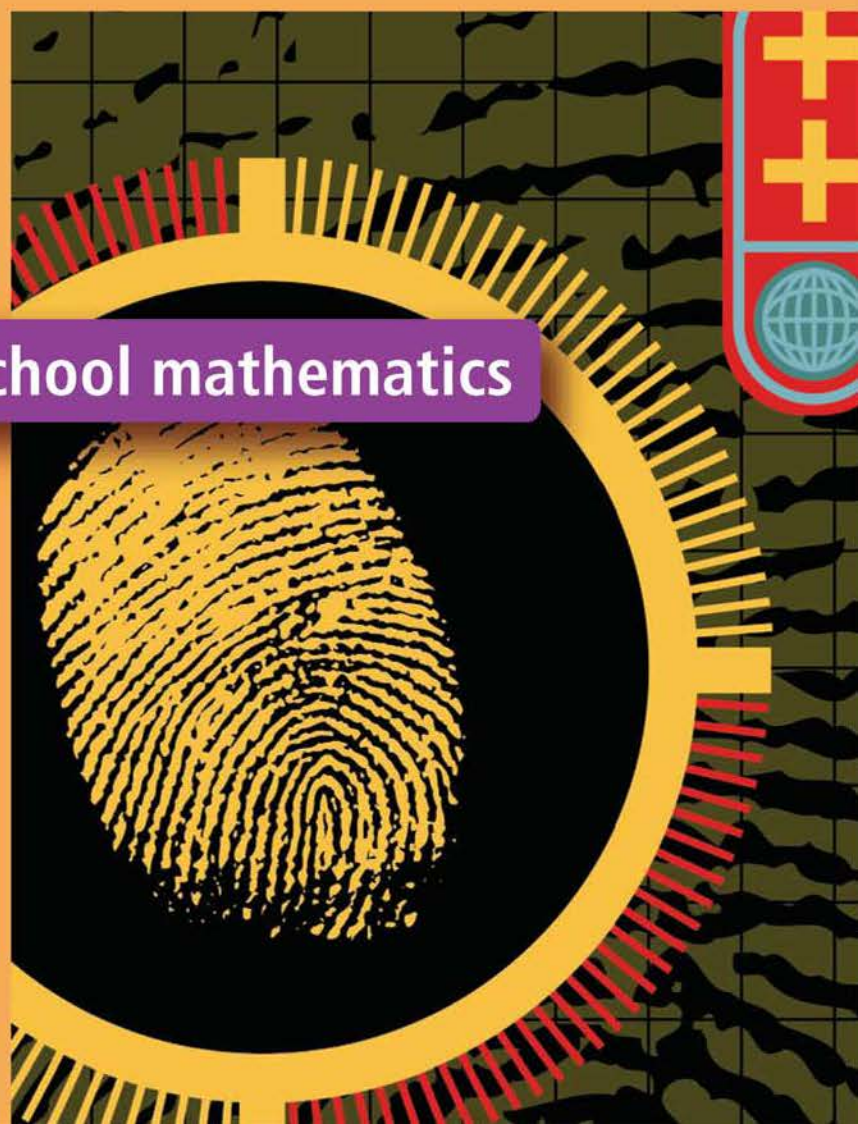
a family's guide:

Fostering your child's success in school mathematics

Prekindergarten–Grade 12



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS



a family's guide

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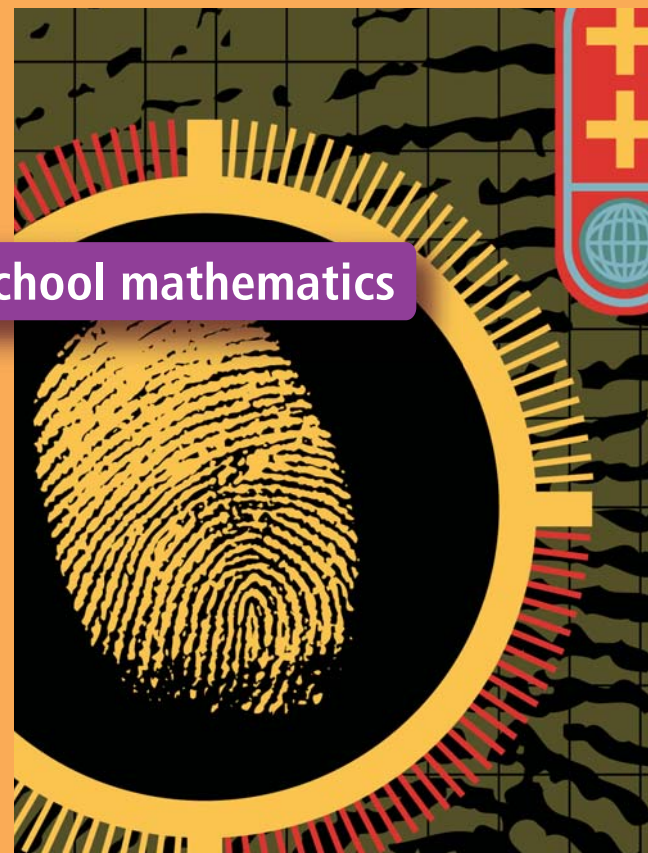
Fostering your child's success in school mathematics

Edited by Amy Mirra



NATIONAL COUNCIL OF
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Prekindergarten–Grade 12



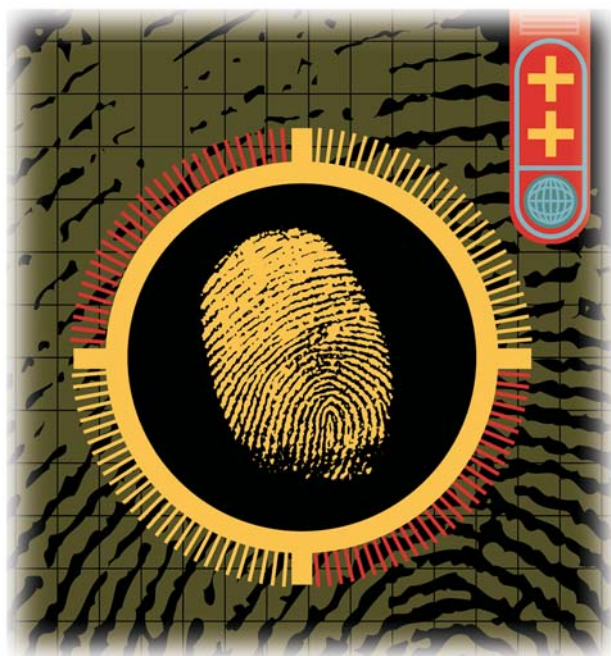
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The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership, and professional development to support teachers in ensuring mathematics learning of the highest quality for all students.

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On the Cover

In the cover design for this publication, a camera aperture focuses on a fingerprint. Just as no two fingerprints are exactly alike, no two children are exactly alike in the way they learn and the way they approach mathematics. NCTM's *Principles and Standards for School Mathematics* (2000) views the teaching of mathematics with that central fact in mind. This book looks at those Standards from the perspective of family, understanding that the combined efforts of teacher, student, and family can lead to better mathematics learning for all students.



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Introduction

What does mathematics mean to you? The hope is that your experiences were positive, resulting in an appreciation and enjoyment of mathematics. Unfortunately, we commonly hear adults make such comments as “I can’t do math” or “I don’t like math.” If you share these sentiments, you may remember mathematics as a bunch of rules to memorize, concepts that did not make sense, and assignments unconnected with everyday life.

Regardless of your own experiences, you play an important role in your child’s attitude toward, and success with, mathematics. This guide is intended to help you understand changes in mathematics education and to suggest ways that you can support your child’s learning of mathematics.

The National Council of Teachers of Mathematics (NCTM) wants all students to engage in high-quality mathematics. Over the past decade, NCTM has worked to build school mathematics programs that will prepare students both for today and for a future we can

only imagine. NCTM’s *Principles and Standards for School Mathematics* (2000) presents a vision for mathematics education that “requires solid curricula, knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence” (NCTM 2000, p. 3).

NCTM believes the following precepts:

Mathematical literacy is essential for every child’s future.

- A solid mathematics education is essential for an informed public, our national security, a strong economy, and national well-being.
- All students can be successful in mathematics and should receive a high-quality mathematics education, regardless of gender, ethnicity, or race.
- Teachers should encourage and inspire every student to continue the study of mathematics.

- Developing mathematical proficiency requires a balance and connection between conceptual understanding and procedural and computational proficiency.
- Problem solving and using mathematics to understand our world are integral parts of all mathematics learning.
- Teachers must have a solid knowledge of both mathematics content and teaching methodologies, as well as enjoy and value mathematics.
- Effective programs of teacher preparation and professional development help teachers understand the mathematics they teach, how their students learn that mathematics, and how to help each student learn.
- Improving mathematics education for *all* students requires a commitment from a variety of stakeholders, including teachers, mathematics teacher-leaders, school and district administrators, institutions of higher learning, mathematicians, professional organizations, families, politicians, business and community leaders, and students.

● ● ● ● ● **Mathematics literacy is essential for every child’s future.**



A Different Mathematics Classroom

Your child's mathematics classroom may look quite different from what you remember from your own student days. Instead of straight rows of desks, you may see desks that are pushed together into groups. Instead of the teacher demonstrating a problem on the board and students silently copying the work, you may see students working together to investigate a problem while the teacher acts as a facilitator. Instead of students reciting memorized rules and procedures, you may see students discussing and questioning solutions to problems.

Over the past few decades, educators have greatly improved their understanding of how students learn mathematics. In high-quality elementary, middle, and high school classrooms, students are actively involved in making sense of the mathematics they are learning. Learning is not a passive activity but rather, an active process of building new knowledge from experience and prior knowledge. Think about your own learning experiences. Chances are that you remember those experiences in which you had the opportunity to be an active participant much more than those in which you were passively listening or observing.



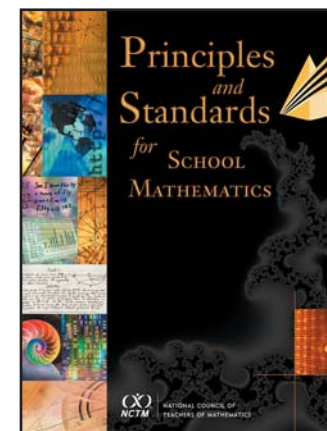
What Will Your Child Be Learning?

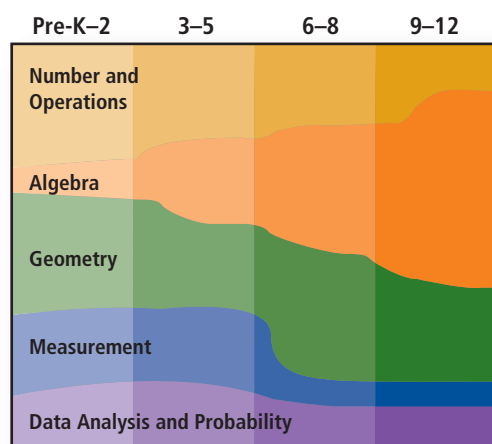
Mathematics is much more than computation problems, equations, and word problems. Mathematics is a way of thinking and exploring. Mathematics is a language that helps us communicate about our world. Most important, mathematics is a tool that we *all* use in our daily lives to solve practical and abstract problems.

A major goal for students today is to learn mathematics with understanding, the same as the goal you have for your child when reading. Not only do you want your child to be able to pronounce and decode words, but you want your child to be able to comprehend what he or she reads. Doing so involves a much higher level of thinking than simply decoding a word. Learning mathematics with understanding is essentially the same thing. No matter how well your child can perform calculations, this ability is not very useful if he or she does not know how to apply these skills. The goal is for your child to develop a lifelong understanding of mathematics that is useful both at home and in the workplace.

Principles and Standards for School Mathematics, published by NCTM in 2000, reflects current

thinking and research on mathematics teaching and learning. From the very beginning, children should be exposed to a variety of mathematical ideas, not just arithmetic. As they get older, their understanding of these ideas will grow and deepen. NCTM's Standards are divided into Content and Process Standards. The Content Standards (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) describe what students should learn. The Process Standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) describe meaningful ways for students to learn that content. Together, these ten Standards define the basic mathematics that all students should have the opportunity to learn, regardless of their decision to move on to the workplace or to higher education after high school. The following are brief descriptions of the Content and Process Standards. A chart listing specific expectations for students in Prekindergarten–grade 2, grades 3–5, grades 6–8, and grades 9–12 can be found at the back of this guide.





Content standards

Number and operations. Young children focus on whole numbers to count objects, compare quantities, and develop an understanding of the base-ten number system. In higher grades, fractions and integers become more prominent. An understanding of numbers allows students to learn and recall computational procedures with ease. Students should use mental methods, estimations, and paper-and-pencil calculations, and should be able to make good decisions about the use of calculators. Students should be able to explain their method, understand that many methods exist, and see the usefulness of methods that are efficient, accurate, and general.

Algebra. Algebraic symbols and the procedures for working with them are essential in mathematical work. Algebra is a style of mathematical thinking for formalizing patterns, functions, and generalizations. Even young children use algebraic reasoning as they study numbers and operations, and as they investigate patterns and relations among sets of numbers. In this standard, the connections of algebra with number and everyday situations are extended in the higher grade bands to include ideas from other standards.

Geometry. Geometry has long been a discipline in which students learn to prove theorems. The Geometry Standard takes a broader view of the power of geometry. Students analyze characteristics of geometric shapes and make mathematical arguments about the relationships among them, as well as use visualization, spatial reasoning, and geometric modeling to solve problems. Geometry is a natural area of mathematics for developing students' reasoning and justification skills.

Measurement. Measurement can serve as a way to connect the different areas of mathematics because it offers opportunities to learn about and apply other areas of mathematics, such as number, geometry, functions, and statistical ideas. The Measurement Standard includes understanding the attributes, units, systems, and processes of measurement, as well as applying appropriate techniques, tools, and formulas to determine measurements.

Data analysis and probability. Reasoning statistically is essential to being an informed citizen and consumer. This standard calls for students to collect, organize, and display relevant data to answer questions they have formulated. It emphasizes learning appropriate statistical methods to analyze data, making inferences and predictions based on data, and understanding and using the basic concepts of probability.



Process standards

Problem solving. By solving problems, students acquire ways of thinking, habits of persistence and curiosity, and confidence in

unfamiliar situations that serve them well outside the mathematics classroom. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that involve a significant amount of effort. They are to be encouraged to explain and discuss their thinking during the problem-solving process so that they can apply and adapt the strategies they develop to other problems and in other contexts.

Reasoning and proof. People who reason and think analytically note patterns, structures, or regularities in mathematical situations. They develop and evaluate mathematical conjectures and proofs, which are formal ways of expressing particular kinds of reasoning and justification. By exploring phenomena, justifying results, and

using mathematical conjectures in all content areas at all grade levels, students should realize and expect that mathematics makes sense.

Communication. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language. Their explanations should include mathematical rationales, not just procedural descriptions. Conversations in which mathematical ideas are explored from multiple perspectives help participants sharpen their thinking and make connections.

Connections. Mathematics is not a collection of separate strands or standards, even though it is often partitioned and presented in this manner. Rather, mathematics is an integrated field of study. When students connect mathematical ideas, their understanding is deeper and more lasting, and they come to view mathematics as a meaningful whole. They see mathematical con-

nections among different mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. Through instruction that emphasizes the interconnectedness of mathematical ideas, students not only learn mathematics but also appreciate the utility of mathematics.

Representations. Mathematical ideas can be represented in a variety of ways: pictures, concrete materials, tables, graphs, numerical and alphabetic symbols, spreadsheet displays, and so on. When students create representations to capture mathematical concepts or relationships, they acquire a set of tools that significantly expand their capacity to model, interpret, and analyze physical and social phenomena in a mathematical way.

“ Together, these ten Standards define the basic mathematics that all students should have the opportunity to learn. ”





What Families Can Do

1

Be positive!

If you have a negative attitude about mathematics, chances are your child will, too. Help your child have a “can do” attitude by praising your child’s efforts as well as her accomplishments. Acknowledge the facts that mathematics can be challenging at times and that persistence and hard work are the keys to success. Relate mathematics learning to other endeavors that require hard work and persistence, such as playing a sport. Struggling at times in mathematics is normal and is actually necessary to, and valuable in, understanding mathematics.

2

Link mathematics with daily life

Every day, people face situations that involve mathematics, such as deciding whether one has enough money to purchase a list of items at the store, reading a map to find out where one is, building a budget, deciding on the shortest route to a destination, developing a schedule, or determining the price of an item on sale. Help your child realize that mathematics is a significant part of everyday life. Suggestions for discussing mathematics with your elementary, middle, or high school child during everyday activities are listed at the end of this section.

Ideas to Help Your Child Succeed in and Enjoy Mathematics

3

Make mathematics fun

Play board games, solve puzzles, and ponder brain teasers with your child. Your child enjoys these kinds of activities while enhancing his mathematical thinking. Point out the mathematics involved, and have your child discuss the strategies he used.

4

Learn about mathematics-related careers

Mathematics is foundational to a wide variety of interesting careers. Research different careers with your child, and find out what she should be doing now to prepare for these options. Help your child understand that the school courses she takes now and the grades she earns will affect her future. One source of information on the many career possibilities that involve mathematics is *Career Ideas for Kids Who Like Math*, by Diane Lindsey Reeves.



5**Have high expectations**

Traditionally, in North America the belief that only some students are capable of learning mathematics has prevailed. For example, tracking has consistently disadvantaged groups of students through classes that concentrate on remediation and do not offer significant mathematical substance. Many students, especially those who are poor, nonnative speakers of English, disabled, female, or members of racial-minority groups, have become victims of low expectations. Today we are guided by a vision of mathematics for all. You would not expect your child not to read; similarly, you should not expect your child not to do mathematics. Your attitude and expectations are crucial to influencing the future opportunities for your child. Communicate high expectations to your child and his teachers, counselors, and administrators. Make sure that your child is getting the same opportunities in mathematics as everyone else, and is taking challenging mathematics classes each year, all the way through high school.

6**Support homework, don't do it!**

Homework is an area that can cause trouble in most households. Relax, and remember whose homework it is. If you take over doing homework for your child, you encourage him to easily give up or seek help when working on a challenging problem. If you start to panic when you do not know how to do the mathematics, you may signal negative thoughts about mathematics to your child. Your child is not likely to be resourceful, persistent, or confident if you react in either of these ways.

Think of yourself as more of a guide rather than your child's teacher. Your role is not only to support her but also to help her take responsibility for herself. You can facilitate your child's homework by asking questions and listening to your child. The simple act of having your child explain something out loud can often help her figure out the problem. Encourage your child to also show all her calculations or a description of her thinking process on paper to support the solution to a problem. This recording gives the student something to look back on, either for review or to spot and fix a mistake. It can also furnish the teacher with useful information related to the student's reasoning and understanding.

“ Every day, people face situations that involve mathematics.”



Keeping students interested in schoolwork as they get older can also become more challenging as friends and outside activities compete for their attention. You may be struggling to remain involved and aware during your child's middle and high school years. You may have a harder time getting to know multiple teachers, you may not be as comfortable with the mathematics your child is learning, or your child may not want you to be as involved as previously. Although your involvement as your child gets older may change, your support is just as important as in the elementary school years. Suggestions to support your child as he or she gets older include the following:

- Help your child develop good study habits and time-management skills (e.g., not waiting until the night before a mathematics test to study for it).
- Give your child latitude in making decisions about schoolwork, but also have high expectations that your child will perform good-quality work in a timely fashion.
- Help your child choose a regular time for studying. Some teens work better immediately after school, whereas others have more energy later in the evening.

Questions to ask your child



- ◆ What new idea did you learn today?
- ◆ What was the most interesting idea or fact you learned?
- ◆ What was the most challenging thing you did today?
- ◆ What did you feel most successful with?
- ◆ What was the funniest thing that happened today?

Questions to ask your child's mathematics teacher



- ◆ What mathematics will be taught in this class?
- ◆ What textbook and other materials will you use this year?
- ◆ Will my child need a calculator, ruler, compass, or other tools?
- ◆ How much homework can we expect, and how long should my child spend on homework each night?
- ◆ May my child work with other students on homework?
- ◆ How should my child make up work after an absence?
- ◆ How do you determine grades?
- ◆ What graded work will be sent home?
- ◆ How often and when will you send home a formal report about my child's performance?
- ◆ How do I schedule a parent-teacher conference?
- ◆ What, if any, standardized tests in mathematics will my child take this year?
- ◆ What can I do to help you?

- Talk openly with your child about the importance of school and any problems or conflicts he may be having.
- Continue to set limits in such areas as the amount of time spent watching television, time spent at a part-time job, and curfew on a school night.
- Encourage your child not to be afraid to ask for help from the teacher or another student. Remind your child that mathematics is more

Doing Mathematics Together— Everyday Situations

Mathematics is all around us! The following activities represent just a sampling of the many everyday situations that involve mathematics and ways you can nurture your child's mathematical development during these activities.

Elementary school years

- Count, count, and count! Young children love to count and will count everything and anything. Encourage your child to count out loud the number of blocks she has, steps climbed, or crackers on her plate.
- Practice classifying by separating toys into sets, such as cars, blocks, or dolls. Ask questions related to size or quantity: Which is larger? Which is largest? Which is smaller? Which is smallest? Do you have more dolls or more bears? Are there fewer dolls or fewer bears?
- Find two and three-dimensional geometric shapes, such as circles and spheres.
- Let your child help set the table. Fold napkins as rectangles one day, then as triangles the next. Find the number of chairs needed to match the places set. Determine the number of knives, forks, and spoons needed.
- Let your child sort the laundry. Before washing, have your child sort the piles by colors or by family members.
- Practice counting and making change. Ask your child to help you

figure out how much money you have in your pocket. Let your child pick out the paper currency and change needed when making a purchase, and have your child tell you how much change you should get back.

- Use sharing to reinforce division concepts and fraction skills. How many cookies will each child get if three children need to share nine cookies? What if one cookie is left over and three children want to share it? How can we cut the birthday cake so we can feed at least twenty people?
- Incorporate measuring during everyday activities, such as cooking, gardening, crafts, or home-improvement projects. Practice measuring things with a ruler, yardstick, tape measure, measuring cup, and scale.
- Use the kitchen to reinforce mathematics concepts and skills. Your child can practice sorting by helping put the groceries away and can practice measuring ingredients by helping cook meals. Older children can determine how to adjust the ingredients to halve or double the recipe. After meals, your child can practice spatial reasoning skills by determining the appropriate size of containers to use for leftovers.
- Numbers are all around us! Look for numbers in the environment (e.g., addresses, sports statistics, weather forecasts, license plates, prices), and talk about what they mean and how they are used.
- Keep charts or graphs to help your child organize information and keep track of data. A child who is saving his allowance to buy an item might create a chart or graph to show how much he can save

“ Mathematics is all around us. ”

- Open a savings account. Work with your child to keep track of deposits, withdrawals, and interest and to compare this record with the monthly bank statement. Research with your child other investment options, such as certificates of deposit (CDs) or money market funds.
- Encourage a child who is a sports enthusiast to keep track of scores and statistics. She can collect data from the newspaper or Internet in a notebook and use these data to make predictions about future performance. She can graph changes over time, for example, the average points scored per game for a favorite basketball player. Finally, your child might keep track of her own performance if she is involved in a sport.
- Turn situations in your household into indirect lessons about money and the importance of wise spending. For example, if you are in the middle of refinancing your home or getting a home equity loan, talk to your child about the process and the outcome. If you are getting a

loan for a new car, talk to your child about the pros and cons of financing for three years versus five years. Visit the Web site www.themint.org for other ideas on helping your middle or high school child understand money and finance,

- Develop a chart with your college-bound child to compare the tuition, room, and board of various colleges, along with opportunities for scholarships or other forms of financial aid. Estimate the total cost including interest for school loans.

Questions ...

Adapted from NCTM's Figure This! Math Challenges for Families "Family Corner" online brochures



Conclusion



Family involvement is an essential element for a student's success in mathematics. You are one of your child's most valuable resources. NCTM applauds your continued efforts to stay involved and learn about ways to support your child's mathematical development. Together with families, educators can provide all students with the highest quality of mathematics education.

**‘ Involvement
is an essential
element. ’**



Resources from NCTM

Purchase resources online at www.nctm.org/catalog, or call customer service at (800) 235-7566 to place an order.



over a certain amount of time. Ask such questions as, How much money do you have now? How much more do you need to save? How long will it take you to save the money?

- Have your child tell you the prices of items while grocery shopping. Older children can help you decide on items that are the better buy or estimate the total bill before paying.
- Set up a play store, or actually sell some products at a garage sale or lemonade stand. Make change, and calculate profit or loss.
- Practice estimation. Adults use estimation rather than find an exact answer as much as 80 percent of the time. Talk to your child about situations in which you use estimation. Ask your child to come up with an estimate, and discuss his strategy. Share your own estimate and strategy. Compare your estimates with the actual answer to determine whether your estimates were reasonable.

Middle and high school years

- Read schedules, such as the television guide and bus transportation schedules, for information.
- Discuss charts, tables, and graphs from the newspaper and magazines. Ask your child to explain what the data show. In a graph that



shows what children like to do before bedtime (watch television, play games on the computer, read, etc.), you might ask such questions as “More than half of children like to do what activity before bedtime?”

- Look at a map with your child to figure out how long a trip will take and what time you should leave in order to arrive on time. Calculate the gasoline mileage, and estimate the total cost of gasoline for long trips.
- Give your child a budget and the responsibility for purchasing clothing or other items. Encourage your child to compare prices and note the savings for items that are on sale. For example, how much would be saved by purchasing the shoes that are 30 percent off?

NCTM Publications

Beyond the Classroom: Linking Mathematics Learning with Parents, Communities, and Business and Industry. Teaching Children Mathematics Focus Issue. February 1998.

80 pp., Stock #674

Elementary School Mathematics: What Parents Should Know about Problem Solving/Estimation by Barbara J. Reys.

20 pp., 1999, Stock #701

How to Use Children's Literature to Teach Mathematics.

75 pp., 1992, Stock #428

Involving Families in School Mathematics: Readings from "Teaching Children Mathematics," "Mathematics Teaching in the Middle School," and "Arithmetic Teacher."

74 pp., 2000, Stock #739

Showcasing Mathematics for the Young Child.

192 pp., 2004, Stock #12662

The Wonderful World of Mathematics: A Critically Annotated List of Children's Books in Mathematics.

355 pp., 1998, Stock #673

The Young Child and Mathematics.

186 pp., 2000, Stock #757

NCTM Web Resources

Families Page: www.nctm.org/families

Lessons, activities, NCTM Standards, Council news, and related resources

Figure This! www.figurethis.org

Activities and math challenges for middle-grades students and families; some challenges also available in Spanish; tips for parents in the Family Corner

Illuminations: www.illuminations.nctm.org

Lessons based on NCTM's Standards, reviewed Web sites, and interactive tools for students to explore, learn, and apply mathematics

Number and Operations

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Pre-K–2

Expectations

In prekindergarten through grade 2 all students should—

Grades 3–5

Expectations

In grades 3–5 all students should—

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

- count with understanding and recognize “how many” in sets of objects;
- use multiple models to develop initial understandings of place value and the base-ten number system;
- develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections;
- develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers;
- connect number words and numerals to the quantities they represent, using various physical models and representations;
- understand and represent commonly used fractions, such as $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$.

- understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals;
- recognize equivalent representations for the same number and generate them by decomposing and composing numbers;
- develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers;
- use models, benchmarks, and equivalent forms to judge the size of fractions;
- recognize and generate equivalent forms of commonly used fractions, decimals, and percents;
- explore numbers less than 0 by extending the number line and through familiar applications;
- describe classes of numbers according to characteristics such as the nature of their factors.

Understand meanings of operations and how they relate to one another

- understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations;
- understand the effects of adding and subtracting whole numbers;
- understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.

- understand various meanings of multiplication and division;
- understand the effects of multiplying and dividing whole numbers;
- identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems;
- understand and use properties of operations, such as the distributivity of multiplication over addition.

Compute fluently and make reasonable estimates

- develop and use strategies for whole-number computations, with a focus on addition and subtraction;
- develop fluency with basic number combinations for addition and subtraction;
- use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.

- develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30×50 ;
- develop fluency in adding, subtracting, multiplying, and dividing whole numbers;
- develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results;
- develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience;
- use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals;
- select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tool.

Number and Operations

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Understand numbers, ways of representing numbers, relationships among numbers, and number systems

Understand meanings of operations and how they relate to one another

Compute fluently and make reasonable estimates

Grades 6–8

Expectations

In grades 6–8 all students should—

- work flexibly with fractions, decimals, and percents to solve problems;
- compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line;
- develop meaning for percents greater than 100 and less than 1;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;
- use factors, multiples, prime factorization, and relatively prime numbers to solve problems;
- develop meaning for integers and represent and compare quantities with them.

- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals;
- understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.

- select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods;
- develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use;
- develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Grades 9–12

Expectations

In grades 9–12 all students should—

- develop a deeper understanding of very large and very small numbers and of various representations of them;
- compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions;
- understand vectors and matrices as systems that have some of the properties of the real-number system;
- use number-theory arguments to justify relationships involving whole numbers.

- judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities;
- develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices;
- develop an understanding of permutations and combinations as counting techniques.

- develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- judge the reasonableness of numerical computations and their results.

Algebra

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Understand patterns, relations, and functions

Represent and analyze mathematical situations and structures using algebraic symbols

Use mathematical models to represent and understand quantitative relationships

Analyze change in various contexts

Pre-K–2

Expectations

In prekindergarten through grade 2 all students should—

- sort, classify, and order objects by size, number, and other properties;
- recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another;
- analyze how both repeating and growing patterns are generated.

- illustrate general principles and properties of operations, such as commutativity, using specific numbers;
- use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.

- model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.

- describe qualitative change, such as a student's growing taller;
- describe quantitative change, such as a student's growing two inches in one year.

Grades 3–5

Expectations

In grades 3–5 all students should—

- describe, extend, and make generalizations about geometric and numeric patterns;
- represent and analyze patterns and functions, using words, tables, and graphs.

- identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers;
- represent the idea of a variable as an unknown quantity using a letter or a symbol;
- express mathematical relationships using equations.

- model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

- investigate how a change in one variable relates to a change in a second variable;
- identify and describe situations with constant or varying rates of change and compare them.

<i>Instructional programs from prekindergarten through grade 12 should enable all students to—</i>	Grades 6–8 Expectations In grades 6–8 all students should—	Grades 9–12 Expectations In grades 9–12 all students should—
Understand patterns, relations, and functions	• represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules; • relate and compare different forms of representation for a relationship; • identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.	• generalize patterns using explicitly defined and recursively defined functions; • understand relations and functions and select, convert flexibly among, and use various representations for them; • analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior; • understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions; • understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions; • interpret representations of functions of two variables.
Represent and analyze mathematical situations and structures using algebraic symbols	• develop an initial conceptual understanding of different uses of variables; • explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope; • use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships; • recognize and generate equivalent forms for simple algebraic expressions and solve linear equations.	• understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; • write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases; • use symbolic algebra to represent and explain mathematical relationships; • use a variety of symbolic representations, including recursive and parametric equations, for functions and relations; • judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
Use mathematical models to represent and understand quantitative relationships	• model and solve contextualized problems using various representations, such as graphs, tables, and equations.	• identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships; • use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts; • draw reasonable conclusions about a situation being modeled.
Analyze change in various contexts	• use graphs to analyze the nature of changes in quantities in linear relationships.	• approximate and interpret rates of change from graphical and numerical data.

Geometry

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Apply transformations and use symmetry to analyze mathematical situations

Use visualization, spatial reasoning, and geometric modeling to solve problems

Pre-K–2

Expectations

In prekindergarten through grade 2 all students should—

- recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;
- describe attributes and parts of two- and three-dimensional shapes;
- investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.

- describe, name, and interpret relative positions in space and apply ideas about relative position;
- describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance;
- find and name locations with simple relationships such as “near to” and in coordinate systems such as maps.

- recognize and apply slides, flips, and turns;
- recognize and create shapes that have symmetry.

- create mental images of geometric shapes using spatial memory and spatial visualization;
- recognize and represent shapes from different perspectives;
- relate ideas in geometry to ideas in number and measurement;
- recognize geometric shapes and structures in the environment and specify their location

Grades 3–5

Expectations

In grades 3–5 all students should—

- identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes;
- classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids;
- investigate, describe, and reason about the results of subdividing, combining, and transforming shapes;
- explore congruence and similarity;
- make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.

- describe location and movement using common language and geometric vocabulary;
- make and use coordinate systems to specify locations and to describe paths;
- find the distance between points along horizontal and vertical lines of a coordinate system.

- predict and describe the results of sliding, flipping, and turning two-dimensional shapes;
- describe a motion or a series of motions that will show that two shapes are congruent;
- identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.

- build and draw geometric objects;
- create and describe mental images of objects, patterns, and paths;
- identify and build a three-dimensional object from two-dimensional representations of that object;
- identify and draw a two-dimensional representation of a three-dimensional object;
- use geometric models to solve problems in other areas of mathematics, such as number and measurement;
- recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.

Geometry

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships

Specify locations and describe spatial relationships using coordinate geometry and other representational systems

Apply transformations and use symmetry to analyze mathematical situations

Use visualization, spatial reasoning, and geometric modeling to solve problems

Grades 6–8

Expectations

In grades 6–8 all students should—

- precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;
- understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects;
- create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

- use coordinate geometry to represent and examine the properties of geometric shapes;
- use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.

- describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling;
- examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

- draw geometric objects with specified properties, such as side lengths or angle measures;
- use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume;
- use visual tools such as networks to represent and solve problems;
- use geometric models to represent and explain numerical and algebraic relationships;
- recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Grades 9–12

Expectations

In grades 9–12 all students should—

- analyze properties and determine attributes of two- and three-dimensional objects;
- explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them;
- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- use trigonometric relationships to determine lengths and angle measures.

- use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations;
- investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

- understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices;
- use various representations to help understand the effects of simple transformations and their compositions.

- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;
- visualize three-dimensional objects from different perspectives and analyze their cross sections;
- use vertex-edge graphs to model and solve problems;
- use geometric models to gain insights into, and answer questions in, other areas of mathematics;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Measurement

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Understand measurable attributes of objects and the units, systems, and processes of measurement

Apply appropriate techniques, tools, and formulas to determine measurements

Pre-K–2

Expectations

In prekindergarten through grade 2 all students should—

- recognize the attributes of length, volume, weight, area, and time;
- compare and order objects according to these attributes;
- understand how to measure using nonstandard and standard units;
- select an appropriate unit and tool for the attribute being measured.

- measure with multiple copies of units of the same size, such as paper clips laid end to end;
- use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meterstick;
- use tools to measure;
- develop common referents for measures to make comparisons and estimates.

Grades 3–5

Expectations

In grades 3–5 all students should—

- understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute;
- understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems;
- carry out simple unit conversions, such as from centimeters to meters, within a system of measurement;
- understand that measurements are approximations and understand how differences in units affect precision;
- explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way.

- develop strategies for estimating the perimeters, areas, and volumes of irregular shapes;
- select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles;
- select and use benchmarks to estimate measurements;
- develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms;
- develop strategies to determine the surface areas and volumes of rectangular solids.

Instructional programs from prekindergarten through grade 12 should enable all students to—

Grades 6–8

Expectations

In grades 6–8 all students should—

Understand measurable attributes of objects and the units, systems, and processes of measurement

- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system;
- understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.

Apply appropriate techniques, tools, and formulas to determine measurements

- use common benchmarks to select appropriate methods for estimating measurements;
- select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision;
- develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes;
- develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders;
- solve problems involving scale factors, using ratio and proportion;
- solve simple problems involving rates and derived measurements for such attributes as velocity and density.

Grades 9–12

Expectations

In grades 9–12 all students should—

- make decisions about units and scales that are appropriate for problem situations involving measurement.

- analyze precision, accuracy, and approximate error in measurement situations;
- understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;
- apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations;
- use unit analysis to check measurement computations.

Data Analysis and Probability

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

Select and use appropriate statistical methods to analyze data

Develop and evaluate inferences and predictions that are based on data

Understand and apply basic concepts of probability

Pre-K–2

Expectations

In prekindergarten through grade 2 all students should—

- pose questions and gather data about themselves and their surroundings;
- sort and classify objects according to their attributes and organize data about the objects;
- represent data using concrete objects, pictures, and graphs.

- describe parts of the data and the set of data as a whole to determine what the data show.

- discuss events related to students' experiences as likely or unlikely.

Grades 3–5

Expectations

In grades 3–5 all students should—

- design investigations to address a question and consider how data-collection methods affect the nature of the data set;
- collect data using observations, surveys, and experiments;
- represent data using tables and graphs such as line plots, bar graphs, and line graphs;
- recognize the differences in representing categorical and numerical data.

- describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed;
- use measures of center, focusing on the median, and understand what each does and does not indicate about the data set;
- compare different representations of the same data and evaluate how well each representation shows important aspects of the data.

- propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions.

- describe events as likely or unlikely and discuss the degree of likelihood using such words as *certain*, *equally likely*, and *impossible*;
- predict the probability of outcomes of simple experiments and test the predictions;
- understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.

Instructional programs from prekindergarten through grade 12 should enable all students to—

Grades 6–8

Expectations

In grades 6–8 all students should—

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them

- formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population;
- select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.

Select and use appropriate statistical methods to analyze data

- find, use, and interpret measures of center and spread, including mean and interquartile range;
- discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.

Develop and evaluate inferences and predictions that are based on data

- use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken;
- make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit;
- use conjectures to formulate new questions and plan new studies to answer them.

Understand and apply basic concepts of probability

- understand and use appropriate terminology to describe complementary and mutually exclusive events;
- use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations;
- compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

Grades 9–12

Expectations

In grades 9–12 all students should—

- understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each;
- know the characteristics of well-designed studies, including the role of randomization in surveys and experiments;
- understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term *variable*;
- understand histograms, parallel box plots, and scatterplots and use them to display data;
- compute basic statistics and understand the distinction between a statistic and a parameter.

- for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics;
- for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools;
- display and discuss bivariate data where at least one variable is categorical;
- recognize how linear transformations of univariate data affect shape, center, and spread;
- identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

- use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions;
- understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference;
- evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions;
- understand how basic statistical techniques are used to monitor process characteristics in the workplace.

- understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases;
- use simulations to construct empirical probability distributions;
- compute and interpret the expected value of random variables in simple cases;
- understand the concepts of conditional probability and independent events;
- understand how to compute the probability of a compound event.

Problem Solving

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Communication

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely

Connections

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

STANDARD

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

Source: *Principles and Standards for School Mathematics* (Reston, Va.: NCTM 2000, pp. 391–402)

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